

# Homework 8

*Due: Wednesday, April 19*

All homeworks are due at 11:59 PM on Gradescope.

**Please do not include any identifying information about yourself in the handin, including your Banner ID.**

Be sure to fully explain your reasoning and show all work for full credit.

## Problem 1

- a. For each of the following pairs of events, identify whether they are independent and justify why or why not.
  - i. When flipping a fair coin three times:
    - the first coin is a tails
    - there is a run of exactly two heads (i.e. two, but not three, heads are flipped in a row)
  - ii. When generating a 0/1 string of length 5:
    - the 3rd digit is a 1
    - there are at least two 0s
  
- b. Suppose you have a bag of 4 balls. One ball is red, one ball is yellow, one ball is blue, and one ball is **red, yellow, AND blue**. Define  $X_1$  as picking a red ball, define  $X_2$  as picking a blue ball, and define  $X_3$  as picking a yellow ball.
  - i. Are the events  $X_1$ ,  $X_2$ , and  $X_3$  pairwise independent?
  - ii. Are the events  $X_1$ ,  $X_2$ , and  $X_3$  triplewise independent?

## Problem 2

- a. Jen and Carmen are playing with a spinner numbered from 1 to 210, and experimenting with different rules. For this round, Jen chose to win when the spinner lands on a number that is not a multiple of 2, 3, or 7. What is the probability that Jen wins? (In other words, what is the probability that the number the spinner lands on does not have 2, 3, or 7 as prime factors?)
- b. Jen and Carmen started choosing increasingly unfair rules for their spinner game, and got into a big fight. Their parents took away their spinner, but now they have found ten blocks with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 on each of them. In true child fashion, they have found a way around the rules. Now, they will draw the blocks randomly from a bag and line them up in the order they were chosen. Carmen wins if their sequence contains the sequence 024 or the sequence 456. (Note that this is an inclusive “or.”) What is the probability Carmen wins?

*P.S. This is all theoretical. In reality, since they are sitting on opposite sides of the sequence of blocks, they will perceive the sequences as the reverse of what the other perceives, and another fight is inevitable.*

## **Problem 3**

This problem is a Lean question!

This homework question can be found by navigating to `BrownCs22/Homework/Hw7.lean` in the directory browser on the left of your screen in Gitpod. The comment at the top of that file provides more detailed instructions.



## Problem 4 (Mind Bender — *Extra Credit*)

This is a plant-themed version of a popular puzzle that made its rounds through math and computer science circles back in 2001:

Imagine a game with three players. Each player has either a rose or a tulip taped to their head, determined independently and uniformly at random, i.e. each player has 50/50 odds of receiving a rose or a tulip. Each player can see the flowers on the other two players heads, but cannot see their own.

After observing each other's flowers, players *simultaneously* guess their flower type or pass (choose not to guess). There is no communication of any form during the game, but the players may devise a strategy ahead of time.

The players collectively win the game if at least one player guesses the type of flower on their head correctly, and no players guess incorrectly. For instance, if one player guesses correctly and the others pass, then everyone wins. On the other hand, if two players guess correctly, but the last player guesses incorrectly, then everyone loses.

One strategy, for example, would be to have player one always guess tulip and have the other two players always pass. This strategy wins 50% of the time.

Devise a strategy that maximizes the players' odds of winning this game. What are their odds using this strategy? Show how you computed this value.