Homework 2

Due: Wednesday, February 15

All homeworks are due at 11:59 PM on Gradescope.

Please do not include any identifying information about yourself in the handin, including your Banner ID.

Be sure to fully explain your reasoning and show all work for full credit.

LaTeX tips

As you begin using LaTeX, here are some useful commands that might come in handy for this homework:

`\Pow`	$\mathcal{P}$	`\cap`	$\cap$	`\cup`	$\cup$
`\star`	$\star$	`\overline{A}`	$\overline{A}$	`\emptyset`	$\emptyset$
`\times`	$\times$	`\setminus`	$\setminus$	`\subseteq`	$\subseteq$
`\forall`	$\forall$	`\exists`	$\exists$	`\land`	$\land$
`\lor`	$\lor$	`\lnot`	$\lnot$	`\to`	$\to$

A note that LaTeX is *not* required until **Homework 3**.

Problem 1

Translate the sentences below into formulas of first-order logic. Use the following symbols. Note: the sets listed here can be used as *domains* of quantification. That is, you could write $\forall x : P, \ldots$ to quantify over all plants. You should not use any other domains of quantification.

- Sets:
  - P: the set of plants
- Predicates:
  - $G(x)$: "x is a geranium"
  - $L(x)$: "x is a lily"
  - $T(x, y)$ "x and y are planted together in the same pot"

  - Familiar mathematical symbols like $<$, $\leq$, $=$ have their normal meanings

- Functions:
  - $f(x)$: the number of flowers on x
  - $n(x)$: the plant nearest to x (that is not x itself)

- Constants:
  - g: Edith's favorite geranium

a. A geranium and a lily are planted in the same pot.

b. Some geranium has more flowers than every lily.

c. The plant nearest to Edith's favorite geranium is also a geranium, but is planted in a different pot.

d. No plant is both a geranium and a lily.

e. Explain your answer to part b. above. Suppose that you had to argue that this statement was true. How would you justify this claim? Think in terms of the proof rules that we have discussed in lecture.

Problem 2

Let $A = \{\varnothing, 3, \text{“plant”}\}$, $B = \{9, 7, 6\}$, $C = \{3, 9, 6\}$, and $D = \{10i \mid i \in \mathbb{Z}, i \in [1, 9]\}$.

Find the cardinalities of the following sets. You only need to justify b, i, and j.

a. A

b. $\{A, C, 3\}$

c. $B \cup C$

d. $A \cap C$

e. $C \setminus B$

f. $\mathcal{P}(B)$

g. $\mathcal{P}(\mathcal{P}(B))$

h. $B \times D$

i. $\mathcal{P}(A \times A)$

j. $\mathcal{P}(\mathcal{P}(A) \times D)$

Note: Your answer may be given as a power of 2.

Problem 3

This problem is a Lean question!

This homework question can be found by navigating to `BrownCs22/Homework/Hw2.lean` in the directory browser on the left of your screen in Gitpod. The comment at the top of that file provides more detailed instructions.

You will submit your solution to this problem separately from the rest of the assignment. Once you have solved the problem, copy the entire file to your computer, and upload it to Gradescope.

Problem 4 (Mind Bender — *Extra Credit*)

Recall that we can create sets with *descriptions* using set-builder notation, like

$$A = \{x \in \mathbb{R} \mid x^2 > 1\}.$$

We would read this as "A is the set of x in the real numbers *such that* $x^2 > 1$." The condition that $x^2 > 1$ is a condition we place on this set to define it. Another example could be

$$T = \{t \mid t \text{ is a kind of tree}\}.$$

in which $T = \{\text{maple}, \text{elm}, \text{oak}, \dots\}$. This is an example of a description that is in natural language. One might consider if *every* description is a valid description.

Consider the description "X does not contain itself".

Example

For example, let

$$T = \{t \mid t \text{ is a kind of tree}\}.$$

be the set of all kinds of trees. This set T does not contain itself ($T \notin T$) since the set of all kinds of trees is not a kind of tree.

However, let

$$U = \{t \mid t \text{ is } \textit{not} \text{ a kind of tree}\}.$$

be the set of everything that *isn't* a kind of tree. Well, U is a set of things, that clearly isn't a kind of tree, so $U \in U$ and we say U "contains itself".

Let's build a set with this specific description. Let S be the set that contains all sets that do not "contain themselves". That is, we define it to be

$$S = \{X \text{ is a set} \mid X \notin X\}$$

A set such as U *would not* be in S, as U contains itself. However, a set such as T *would* be in S, as T does not contain itself.

a. Show that the assumption that S is a member of S leads to a contradiction.

b. Show that the assumption that S is not a member of S leads to a contradiction.

c. What do these contradictions suggest about how we can or cannot define a set? This paradox is called *Russell's Paradox*. Are there any ways to resolve these contradictions in set theory? Do some research and cite at least one source.[1]

[1] This is an open ended question, and there is no right answer! You should demonstrate to us that you have an understanding of what is going on.