Homework 10

Due: Wednesday, May 5th

All homeworks are due at 11:59 PM on Gradescope.

Please do not include any identifying information about yourself in the handin, including your Banner ID.

Be sure to fully explain your reasoning and show all work for full credit.

LaTeX tips

Here are some LaTeX commands that might help you:

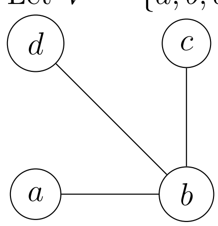
`\overline{G}` $\overline{G}$

You may draw your graphs by hand or any method you would like, and include images using `\includegraphics[scale=.5]{graph.png}`. If you are feeling brave, look into drawing graph diagrams in LaTeX using the `tikz` package!

Problem 1

Let $\mathbb{E}$ denote the set of edges of a complete graph with vertices V. Suppose G is a graph with vertex set $V(G) = V$ and edges $E(G) \subseteq \mathbb{E}$. The complement of G, $\overline{G}$, is the graph where $V(\overline{G}) = V$ and $E(\overline{G}) = \mathbb{E} \setminus E(G)$.

a. Let $V = \{a, b, c, d\}$. Draw the complement of the graph on V shown below.



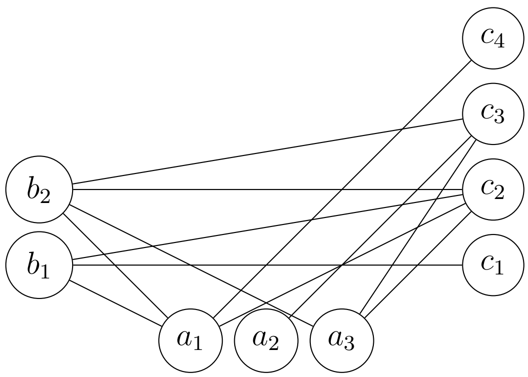
b. It is possible to have a graph G where both G and $\overline{G}$ are connected. Give an example of such a graph. Draw both G and $\overline{G}$. Recall, a graph is **connected** if there is a simple path between each pair of vertices (that is, all vertices are connected).

c. Suppose G is a graph on 9 vertices such that G has an Eulerian tour, and both G and $\overline{G}$ are connected. Prove that $\overline{G}$ must have an Eulerian tour.

Problem 2

In this problem, we are interested in graphs G with the following structure: the vertex set $V(G)$ is partitioned into three nonempty disjoint sets $V(G) = V_1 \cup V_2 \cup V_3$, and edges only connect vertices in distinct partitions. If a graph G has this structure, we will call it *3-isolated*.

Here is an example of a 3-isolated graph with $V_1 = \{a_1, a_2, a_3\}, V_2 = \{b_1, b_2\}, V_3 = \{c_1, c_2, c_3, c_4\}$. Notice that there are no edges connecting any of the a vertices to each other, and similarly for the b and c vertices.



a. Consider the vertex set $V_1 = \{a_1, a_2, a_3\}, V_2 = \{b_1, b_2\}, V_3 = \{c_1, c_2, c_3, c_4\}$ shown above. How many possible 3-isolated graphs can be drawn on this vertex set?

b. I make the following conjecture: on any vertex set V_1, V_2, V_3 with $|V_i| \geq 2$ for each i (in other words, each set of vertices has at least 2 vertices), it is impossible to draw a 3-isolated graph with a Hamiltonian cycle.

   Prove or disprove my conjecture!