CSCI0220 Practice Midterm

Please read in full:

The following practice exam will look somewhat different from the one you receive on exam day. This practice exam is derived from last year's midterm.

The in-class exam this year will cover similar information and be of similar difficulty in questions, but will be designed and formatted for an in-class exam. *You should not draw any conclusions about the length of the exam from this practice exam.*

It's a good idea to try the exam first by yourself, and then read over the solutions.

Problem 1

a. Prove using the set-element method that for sets Y, B, and C with the same universal set U:

$$(Y \setminus B) \cap (Y \setminus C) = Y \setminus (B \cup C).$$

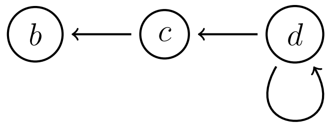
b. The following proposition is false.

> **Not a theorem**. For any sets A, B, C, and D, define
>
> $$L = (A \cup B) \times (C \cup D)$$
> $$R = (A \times C) \cup (B \times D)$$
>
> Then $L = R$.

Show a counterexample to this proposition: Give four sets A, B, C, D for which this equality is false. Explicitly give an example of an element that is in either L or R but is not in the other.

Problem 2

a. This diagram shows a relation R on the set $\{b, c, d\}$, where xRy if and only if there is an arrow from x to y.



By *adding* arrows to this diagram, without removing any existing arrows, turn the relation R into one that is both reflexive and transitive.

Is your relation also symmetric? Explain why or why not!

b. We now consider the relation is_gcd : $(\mathbb{Z} \times \mathbb{Z}) \to \mathbb{Z}^+$. This is a relation between pairs of integers and positive integers. is_gcd$((a, b), g)$ holds exactly when g is the greatest common divisor of a and b.

i. Is is_gcd an injective relation? Is it surjective? For each of your answers, give a proof or a counterexample.

ii. The Euclidean algorithm can be said to "compute" the is_gcd relation. Given input (a, b) the algorithm should produce a g such that is_gcd$((a, b), g)$ holds. Showing the intermediate steps, use the Euclidean algorithm to compute the gcd of 68 and 28.

(Note: you do *not* need to use the extended Euclidean algorithm.)

Problem 3

Tyler, Ryan, and Joseph are bank robbers who are very precise about the amount of money they steal: Each robbery, they steal precisely $n^3 - n$ dollars for some natural number n.

Use induction to prove that they are always able to divide the money they steal evenly among the three of them.

(Note: even if you are unable to finish the problem, show how you would set up your proof!)

Problem 4

a. Let x be a propositional formula with Boolean inputs a, b and c and the following truth table:

a	b	c	x
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	F

Write a formula in disjunctive normal form that is equivalent to the formula x. Explain how you arrived at your answer.

b. Is the formula $\neg q \to ((p \vee q) \to p)$ unsatisfiable, satisfiable (but not valid), or valid? Again, explain your answer!

