CS22 Exam 1: Math Foundations (Fall 2025)

ANSWER KEY ANSWER KEY ANSWER KEY ANSWER KEY

Please read these directions carefully before you begin!

- This is a *closed-book* exam. You may not use any external resources.
- This exam is for you alone. Do not share it or your solutions with anyone.
- Write your name and Brown ID number on this cover sheet, and your Brown ID on subsequent sheets. This will help us keep track of the pages of your exam while grading anonymously.
- No electronic devices are allowed, including cell phones! Please turn off and put away your phones.
- Solutions will be graded on correctness and clarity. Partial solutions will be graded for partial credit.
- Your work should be yours and yours alone. Please do not look at other people's exams! There are multiple versions of the exam and your neighbors probably do not have the same one as you.
- You should spend about 1 minute per point.
- For this section, there are 5 total pages (including this cover sheet) and 4 questions.
- Please write your answers in the space provided on the exam, and clearly mark your solutions. You may use the backs of the exam pages as scratch paper. *We will not grade answers written on the backs of pages.* If you need additional scratch paper, please request it from the proctoring staff. Do not use your own scratch paper.
- Good luck!

Student name: _____

Brown ID: _____

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1. (4 points) Select exactly one option for each item.

   i. Let $A = \{1, 2, 3\}$. Which of the following is true?

   ✓ $\{2, 3\} \subseteq A$ ○ $\{2, 3\} \in A$ ○ $2 \subseteq A$

   ii. What is $|\mathcal{P}(\{1, 2, 3\})|$?

   ○ 3 ○ 6 ✓ 8

   iii. What is $\{1, 2\} \times \{3\}$?

   ✓ $\{(1, 3), (2, 3)\}$ ○ $\{(3, 1), (3, 2)\}$ ○ $\{1, 2, 3\}$

   iv. Which of the following relations is a (total) function with domain $\{a, b\}$ and codomain $\{1, 2\}$?

   ✓ $\{(a, 1), (b, 1)\}$ ○ $\{(a, 1), (a, 2), (b, 1)\}$ ○ $\{(a, 1)\}$

2. (4 points) Select exactly one option for each item.

   i. Let $f : \mathbb{Z} \to \mathbb{Z}$ be the function $f(x) = 2x$. Then f is:

   √ **injective but not surjective**
   ◯ surjective but not injective
   ◯ bijective
   ◯ neither injective nor surjective

   ii. Let $A = \{1, 2\}$. Which of the following is *false*?

   ◯ $\emptyset \in \mathcal{P}(A)$ ◯ $\{1\} \in \mathcal{P}(A)$ √ $\{1\} \subseteq \mathcal{P}(A)$ ◯ $A \in \mathcal{P}(A)$

   iii. Which of the following relations on $\mathbb{Z}$ is an equivalence relation?

   ◯ $\{(x, y) : x \leq y\}$ √ $\{(x, y) : |x| = |y|\}$ ◯ $\{(x, y) : x - y = 1\}$

   iv. Let $f : A \to B$ be a function where $|A| = 5$ and $|B| = 3$. Which of the following must be true?

   √ f **is not injective** ◯ f is surjective ◯ f is bijective

3. (9 points) Prove by the contrapositive that for any integer n, if n^2 is odd then n is odd.

Solution: Let n be an arbitrary integer. Suppose n is not odd. Thus n is even which means $n = 2k$ for some integer k. Then $n^2 = (2k)^2 = 2 \cdot (2k^2)$. Since $2k^2$ is an integer, this is even and therefore not odd.

4. (8 points) For any natural number n that is at least 1 we have

$$1 + 2 + 4 + 8 + \ldots + 2^{n-1} = 2^n - 1.$$

The below is a proof of this fact by induction with most of the inductive step left blank. Fill in the rest of the inductive step. It might be useful to remember $2 \cdot 2^n = 2^{n+1}$.

The proof is by induction on n starting at 1.

The predicate is $P(n)$: $1 + 2 + 4 + 8 + \ldots + 2^{n-1} = 2^n - 1$.

As a base case for $n = 1$ we have $2^{1-1} = 1 = 2^1 - 1$ so $P(1)$ holds.

For the inductive step fix an arbitrary n and suppose $P(n)$.

Solution: We would like to show $P(n+1)$:

$$1 + 2 + 4 + 8 + \ldots + 2^n = 2^{n+1} - 1$$

Applying $P(n)$, we have the left hand side is

$$\begin{aligned} 1 + 2 + 4 + 8 + \ldots + 2^{n-1} + 2^n &= 2^n - 1 + 2^n \\ &= 2^{n+1} - 1 \end{aligned}$$

Since we have shown the base case and the inductive step, we conclude by induction that $P(n)$ holds for all natural numbers at least 1.