CS22 Exam 1: Logic (Fall 2025)

ANSWER KEY ANSWER KEY ANSWER KEY ANSWER KEY

Please read these directions carefully before you begin!

- This is a *closed-book* exam. You may not use any external resources.
- This exam is for you alone. Do not share it or your solutions with anyone.
- Write your name and Brown ID number on this cover sheet, and your Brown ID on subsequent sheets. This will help us keep track of the pages of your exam while grading anonymously.
- No electronic devices are allowed, including cell phones! Please turn off and put away your phones.
- Solutions will be graded on correctness and clarity. Partial solutions will be graded for partial credit.
- Your work should be yours and yours alone. Please do not look at other people's exams! There are multiple versions of the exam and your neighbors probably do not have the same one as you.
- You should spend about 1 minute per point.
- For this section, there are 5 total pages (including this cover sheet) and 4 questions.
- Please write your answers in the space provided on the exam, and clearly mark your solutions. You may use the backs of the exam pages as scratch paper. *We will not grade answers written on the backs of pages.* If you need additional scratch paper, please request it from the proctoring staff. Do not use your own scratch paper.
- Good luck!

Student name: _____

Brown ID: _____

ANSWER KEY ANSWER KEY ANSWER KEY ANSWER KEY

1. (4 points) Select exactly one option for each item.

   i. Which of the following is a proposition?

   ◯ Please sit down. √ $2 \cdot 2 = 5$. ◯ Ouch!

   ii. Suppose p is false and q is true. Then $p \to q$ is:

   √ **true** ◯ false ◯ neither true nor false

   iii. Which of the following formulas is in disjunctive normal form (DNF)?

   √ $(p \wedge \neg q) \vee (\neg p \wedge r)$ ◯ $(p \vee \neg q) \wedge (\neg p \vee r)$ ◯ $\neg(p \wedge q) \vee r$

   iv. In the expression $\forall x\ (p(x) \to \exists y\ q(x, y)) \wedge r(y)$, the y in "$r(y)$" is:

   ◯ bound √ **free** ◯ neither

2. (4 points) Select exactly one option for each item.

   i. Which of the following is equivalent to $\neg(p \to q)$?

   √ $p \wedge \neg q$ ◯ $\neg p \wedge q$ ◯ $\neg p \to \neg q$ ◯ $p \to \neg q$

   ii. Which of the following is equivalent to $\neg\forall x\, \exists y\, P(x, y)$?

   √ $\exists x\, \forall y\, \neg P(x, y)$ ◯ $\forall x\, \exists y\, \neg P(x, y)$ ◯ $\exists x\, \exists y\, \neg P(x, y)$ ◯ $\forall x\, \forall y\, \neg P(x, y)$

   iii. Which of the following is a valid rule of inference?

   ◯ $\dfrac{\begin{array}{c} q \\ p \to q \end{array}}{\therefore p}$ √ $\dfrac{\begin{array}{c} \neg q \\ p \to q \end{array}}{\therefore \neg p}$ ◯ $\dfrac{\begin{array}{c} \neg p \\ p \to q \end{array}}{\therefore \neg q}$

   iv. Which of the following is *not* satisfiable?

   ◯ $p \wedge \neg q$ √ $(p \vee q) \wedge \neg p \wedge \neg q$ ◯ $\neg p \vee q$

3. Consider the following set and predicates.

- Set:
  - V: all vegetables
- Predicates:
  - $F(x)$: "x is a fruit"
  - $S(x)$: "x has seeds"
  - $T(x, y)$: "x is as tasty as y"

(a) (4 points) Translate each of the below propositions to English.

i. $\forall x \in V\ S(x) \rightarrow T(x, \text{zucchini})$

Solution: Every vegetable with seeds is as tasty as zucchini.

ii. $\neg\exists x \in V\ \forall y \in V\ S(x) \wedge T(x, y)$

Solution: There is no vegetable that has seeds and is as tasty as all other vegetables.

(b) (4 points) Translate the below sentences into first order logic using only the above set and predicates. You should only use the above set as your domain for quantification.

i. Every vegetable that is a fruit has seeds.

Answer: $\forall x \in V\ F(x) \rightarrow S(x)$

ii. If a vegetable is as tasty as all vegetables, then it is a fruit with seeds.

Answer: $\forall x \in V\ (\forall y \in V\ T(x, y)) \rightarrow (F(x) \wedge S(x))$

4. (9 points) Give a logic proof of the conclusion r from premises $q \to r$ and $q \wedge p$ using only the valid rules of inference listed below.

$$\text{(R1)}\ \frac{\begin{array}{c} p \\ p \to q \end{array}}{\therefore q} \qquad\qquad \text{(R2)}\ \frac{p \wedge q}{\therefore p}$$

Solution:

$q \wedge p$	(premise)	(1)
q	(R2 applied to (1))	(2)
$q \to r$	(premise)	(3)
r	(R1 applied to (2), (3))	(4)