CS22 Practice Exam 2: Combinatorics (Fall 2026)

ANSWER KEY ANSWER KEY ANSWER KEY ANSWER KEY

Please read these directions carefully before you begin!

For this section, there are 5 total pages (including this cover sheet) and 4 questions.

Before the Exam

- At the start of the exam, you should have one exam for each section you are doing.
- Please sit with a gap between you and the people on either side (as much as you're able).
- No electronic devices are allowed. Please turn off and put away your phone.
- Write your name and Brown ID number on this cover sheet, and write your Brown ID in the top right of every page. To do this, just flip up the top right corner of each page; do not open the exam or look at the questions. This will help us grade anonymously.
- Do not start the exam until the time starts.

During the Exam

- This is a *closed-book* exam. You may not use any external resources.
- Your work should be yours and yours alone. Please do not look at other people's exams!
- You may do the sections in any order and split the exam time among them as you like.
- As a rough guide, plan to spend about one minute per point.
- We will only grade what is written on the front of the pages. You may use the backs of the pages as scratch paper. However we will not grade answers written on the backs of pages.
- If you run out of space on the front of the page, please ask for extra scratch paper and continue your answer on the front of the scratch paper. If you do this, when you turn in your exam, you must let us know so that we can staple your scratch paper into your exam.
- If you have a question, raise your hand and we will come to you.
- If you need extra scratch paper, raise your hand; do not use your own.
- To use the bathroom, raise your hand first and leave your phone on your desk.

After the Exam

- This exam is for you alone. Do not share it or your solutions with anyone.
- Solutions will be graded on correctness and clarity. Partial correctness will get partial credit.

Good luck!

Student name: _____

Brown ID: _____

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1. (4 points) Select exactly one option for each item.

   i. Recall that $\mathbb{N} = \{1, 2, 3, \ldots\}$. Which of the following is true?
   ◯ $|\mathbb{N}| < |\mathbb{N} \times \mathbb{N}|$ ◯ $|\mathbb{N}| > |\mathbb{N} \times \mathbb{N}|$ √ $|\mathbb{N}| = |\mathbb{N} \times \mathbb{N}|$

   ii. The number of permutations of an n element set is
   ◯ n ◯ $(n-1)!$ √ $n!$ ◯ n^n

   iii. The number of bitstrings of n bits with exactly k ones is
   ◯ $n!$ √ $\binom{n}{k}$ ◯ $\binom{n+k-1}{k-1}$ ◯ k

   iv. For any sets A and B we have that $|A \cup B|$ is
   ◯ $|A| + |B|$ ◯ $|A| + |B| - 2|A \cap B|$ √ $|A| + |B| - |A \cap B|$ ◯ $|A| + |B| + |A \cap B|$

2. (4 points) Select exactly one option for each item below.

   i. Given sets A and B and function $f : A \to B$, if f is not injective then $|A| > |B|$.

   ◯ True √ **False**

   ii. Recall the formula for stars and bars which says that the number of ways to place $k-1$ bars among n stars is $\binom{n+k-1}{k-1}$. If I am supposed to buy 5 donuts and there are 3 varieties, then the number of ways for me to buy donuts is

   ◯ $\binom{8}{3}$ √ $\binom{7}{2}$ ◯ $\binom{8}{2}$ ◯ $\binom{7}{3}$ ◯ $\binom{7}{4}$

   iii. Suppose that n is a positive integer divisible by 12. Let $A = \{4, 8, 12, \ldots, n\}$ be all multiples of 4 up to n and let $B = \{6, 12, 18, \ldots, n\}$ be all multiples of 6 up to n. Then $|A \cup B|$ is

   ◯ $\frac{n}{6}$ ◯ $\frac{n}{2}$ √ $\frac{n}{3}$ ◯ $\frac{n}{12}$

   iv. For any integer $n \geq 1$, among any distinct $n+1$ integers, there are two distinct integers with equal remainder when dividing by n.

   √ **True** ◯ False

3. (8 points) Ellis is again choosing a new exciting password. The requirements for his (case-sensitive) password are that:

   - The password must consist of 5 characters.
   - A valid character is either a lower case letter ("a" through "z" of which there are 26), an upper case letter ("A" through "Z" of which there are 26), the symbol "!" or the symbol "$".
   - The password must include exactly one "!" and one "$" **but these cannot appear next to each other.**

   For example, some valid and distinct passwords are "CS!S$", "!CC$S", "$c!SA"

   Give the number of possible passwords. You do not need to simplify your answer but you should justify your answer.

Solution: We apply the generalized product rule to break this down into first choosing the !, then choosing the $ and then choosing the letters. If we choose ! on the ends of the password then there are 3 choices for $. Otherwise, if we choose ! in the middle 3 characters, there are only 2 choices for $. Thus, the total number of ways to place the ! and $ are $2 \cdot 3 + 3 \cdot 2$. For the remaining 3 characters any of the 52 characters are valid so we get

$$(2 \cdot 3 + 3 \cdot 2) \cdot 52^3 = 12 \cdot 52^3$$

possibilities.

4. (9 points) Recall that $\{0,1\}^n$ is all bitstrings consisting of n bits. For example,

$$\{0,1\}^3 = \{000, 001, 010, 011, 100, 101, 110, 111\}.$$

Prove that for any integer $n \geq 1$ and $A \subseteq \{0,1\}^n$, if $|A| \geq n+2$, then there are two distinct elements of A with the same number of 1s.

Solution: Fix an arbitrary n and $A \subseteq \{0,1\}^n$. Each element of A has either 0 or 1 or ... or n ones in it. Thus, there are $n+1$ options for how many ones an element of A can have. Treating the number of ones as the holes and the elements of A as the things where we place an element of A into a hole depending on its number of ones, since $|A| \geq n+2$ by the pigeonhole principles two elements of A must be in the same hole and therefore have the same number of ones.